

PROPER DISCS IN STEIN MANIFOLDS AVOIDING COMPLETE PLURIPOLAR SETS

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1. INTRODUCTION AND THE RESULTS

Denote by Δ the open unit disc in \mathbb{C} . Recall that a subset Y in a complex manifold X is called *complete pluripolar* if there exists a plurisubharmonic function ρ on X such that $Y = \{z; \rho(z) = -\infty\}$.

In this paper we prove the following result.

Theorem 1.1. *Let X be a Stein manifold of dimension at least 2. Given a closed complete pluripolar set $Y \subset X$, a point $p \in X \setminus Y$ and a vector v tangent to X at p , there exists a proper holomorphic map $f: \Delta \rightarrow X$ such that $f(0) = p$, $f'(0) = \lambda v$ for some $\lambda > 0$ and $f(\Delta) \cap Y = \emptyset$.*

Clearly, every closed complex analytic subset A of a connected Stein manifold X , $A \neq X$, is locally complete pluripolar, that is, for any point $a \in A$ there is an open neighborhood U of a such that $A \cap U$ is complete pluripolar in U . By [Col] every closed locally complete pluripolar set in a Stein manifold is complete pluripolar, thus every closed complex analytic subset is closed complete pluripolar. Therefore our theorem answers the question posed in [FG2] on the existence of proper holomorphic discs in the complements of hypersurfaces.

J. Globevnik [Glo] proved in 2000 that for any point p in a Stein manifold X of dimension at least 2 there exists a proper holomorphic map from the unit disc to X with the point p in its image.

The most general result on avoiding certain sets by proper holomorphic discs was given by H. Alexander [Ale] in 1975: he proved that for a closed polar set $E \subset \mathbb{C}$ there exists a proper holomorphic map $F = (F_1, F_2): \Delta \rightarrow \mathbb{C}^2$ such that $F_1(\Delta) \cap E = \emptyset$. On the other hand, a proper holomorphic disc in \mathbb{C}^2 cannot avoid a non-polar set of parallel complex lines (see [Jul, Tsu, Ale, FG2]). F. Forstnerič and J.

Globevnik [FG2] in 2001 constructed a proper holomorphic disc in \mathbb{C}^2 omitting both coordinate axes and proper holomorphic discs avoiding large real cones in \mathbb{C}^2 . However, it was unknown if the image of a proper holomorphic map from the disc can miss three or more complex lines. Our theorem provides a positive answer to this question since a finite union of complex lines in \mathbb{C}^2 is closed complete pluripolar. Note that closed convex sets in \mathbb{C}^2 which can be avoided by the image of proper holomorphic maps from the disc were characterized in [Dri].

We shall prove the following approximation theorem, which easily implies Theorem 1.1. In fact, Theorem 1.1 will follow directly from Lemma 2.3.

Theorem 1.2. *Let X be a Stein manifold of dimension at least 2 and let $Y \subset X$ be a closed complete pluripolar subset. Let d be a complete metric on X which induces the manifold topology. Assume that $f: \Delta \rightarrow X$ is a holomorphic map such that there is an open subset $V \subset \subset \Delta$ with the property $f(\zeta) \notin Y$ for $\zeta \in \Delta \setminus V$. Given $\epsilon > 0$ there is a proper holomorphic map $g: \Delta \rightarrow X$ satisfying*

- (i) $g(\zeta) \notin Y$ for $\zeta \in \Delta \setminus V$,
- (ii) $d(g(\zeta), f(\zeta)) < \epsilon$ for $\zeta \in V$,
- (iii) $g(0) = f(0)$ and $g'(0) = \lambda f'(0)$ for some $\lambda > 0$.

We will prove Theorem 1.2 in section 2.

Corollary 1.3. *Let X be a Stein manifold of dimension at least 2 and let $Y \subset X$ be a closed complete pluripolar subset. Assume that S is a discrete subset of X such that $S \cap Y = \emptyset$. Then there are proper holomorphic maps $f_n: \Delta \rightarrow X$ such that $f_n(\Delta)$ are pairwise disjoint, $f_n(\Delta)$ avoids Y ($n \in \mathbb{N}$) and $\cup_n f_n(0) = S$.*

Proof. We first note that a finite union of complete pluripolar sets is complete pluripolar, since a finite sum of plurisubharmonic functions is plurisubharmonic. We will also need the fact that a discrete set S in a Stein manifold is complete pluripolar. Namely, by [Col] it is enough to prove that S is locally complete pluripolar, which follows from the fact that S is a complex analytic subset of X .

Let $S = \{s_n; n \in \mathbb{N}\}$. We shall construct the maps f_n inductively. By Theorem 1.1 there is a proper holomorphic map $f_1: \Delta \rightarrow X$ such that $f_1(\Delta) \cap (Y \cup S \setminus \{s_1\}) = \emptyset$ and $f_1(0) = s_1$. Assume that for some

$n \in \mathbb{N}$ we have already constructed proper holomorphic maps $f_j: \Delta \rightarrow X$, $1 \leq j \leq n$, such that $f_j(\Delta)$ are pairwise disjoint, $f_j(\Delta)$ avoids Y and $f_j(\Delta) \cap S = \{s_j\}$ ($1 \leq j \leq n$). By Remmert's proper mapping theorem [Re1, Re2], [Ch2, p. 65] the image of a proper holomorphic map is a closed analytic subset of X and therefore closed complete pluripolar. Thus $Y \cup f_1(\Delta) \cup \dots \cup f_n(\Delta) \cup S \setminus \{s_{n+1}\}$ is closed complete pluripolar. Then by Theorem 1.1 there is a proper holomorphic map $f_{n+1}: \Delta \rightarrow X$ such that

$$f_{n+1}(\Delta) \cap (Y \cup f_1(\Delta) \cup \dots \cup f_n(\Delta) \cup S \setminus \{s_{n+1}\}) = \emptyset$$

and $f_{n+1}(0) = s_{n+1}$. The inductive construction is finished and the proof is complete. \square

Let \mathcal{R} be a bordered Riemann surface. By the theorem of Ahlfors [Ahl], there are inner functions on \mathcal{R} . Recall that a nonconstant continuous function $f: \mathcal{R} \rightarrow \overline{\Delta}$, which is holomorphic on $\mathcal{R} \setminus b\mathcal{R}$, is called an *inner function* (or an *Ahlfors function*) on \mathcal{R} if $|f| = 1$ on $b\mathcal{R}$. Therefore Theorem 1.1 implies the following:

Corollary 1.4. *Let X be a Stein manifold of dimension at least 2 and let $Y \subset X$ be a closed complete pluripolar subset. Given a bordered Riemann surface \mathcal{R} there is a proper holomorphic map $f: \mathcal{R} \setminus b\mathcal{R} \rightarrow X$ such that $f(\mathcal{R} \setminus b\mathcal{R}) \cap Y = \emptyset$.*

2. PROOF OF THEOREM 1.2

As it was observed in [FG2] the methods developed in [FG1, Glo] actually prove the following:

Theorem 2.1. *Let X be a Stein manifold of dimension at least 2 and $\rho: X \rightarrow \mathbb{R}$ a smooth exhaustion function which is strongly plurisubharmonic on $\{\rho > M\}$ for some $M \in \mathbb{R}$. Let $f: \overline{\Delta} \rightarrow X$ be a continuous map which is holomorphic on Δ such that $\rho(f(\zeta)) > M$ for each $\zeta \in b\Delta$. Let d be a complete metric on X which induces the manifold topology. For any numbers $0 < r < 1$, $\epsilon > 0$, $N > M$ and for any finite set $A \subset \Delta$ there exists a continuous map $g: \overline{\Delta} \rightarrow X$, holomorphic on Δ , satisfying*

- (i) $\rho(g(\zeta)) > N$ for $\zeta \in b\Delta$,
- (ii) $\rho(g(\zeta)) > \rho(f(\zeta)) - \epsilon$ for $\zeta \in \overline{\Delta}$,

- (iii) $d(f(\zeta), g(\zeta)) < \epsilon$ for $|\zeta| \leq r$, and
- (iv) $g(\zeta) = f(\zeta)$ and $g'(\zeta) = f'(\zeta)$ for $\zeta \in A$.

In the proof of Theorem 1.2 we also need the following lemma which is a slight generalization of [Ch1, Lemma 1]. Since its proof is essentially the same, we omit it.

Lemma 2.2. *Let X be a Stein manifold and $Y \subset X$ a complete pluripolar set. Let $L_1 \subset L_2 \subset X$ be holomorphically convex compact sets. Then the set $(L_1 \cup Y) \cap L_2$ is holomorphically convex.*

The main tool in the proof of Theorem 1.2 is the following

Lemma 2.3. *Let X be a Stein manifold of dimension at least 2 and let $Y \subset X$ be a closed complete pluripolar subset. Let d be a complete metric on X which induces the manifold topology. Assume that $f: \Delta \rightarrow X$ is a holomorphic map such that there is an open subset $V \subset \subset \Delta$ with the property $f(\zeta) \notin Y$ for $\zeta \in \Delta \setminus V$. Given $\epsilon > 0$ there are a domain Ω , $\{0\} \cup V \subset \subset \Omega \subset \subset \Delta$, conformally equivalent to the unit disc and a proper holomorphic map $g: \Omega \rightarrow X$ with the following properties*

- (i) $g(\zeta) \notin Y$ for $\zeta \in \Omega \setminus V$,
- (ii) $d(g(\zeta), f(\zeta)) < \epsilon$ for $\zeta \in V$,
- (iii) $g(0) = f(0)$ and $g'(0) = f'(0)$.

Proof. One can choose a simply connected domain Ω_1 such that $\{0\} \cup V \subset \subset \Omega_1 \subset \subset \Delta$. By [Hör, Theorem 5.1.6] there is a smooth strongly plurisubharmonic exhaustion function ρ for Stein manifold X . Sard's theorem implies that one can choose a strictly increasing sequence $\{M_n\}$ of regular values of ρ converging to ∞ with M_1 so big that $\rho(f(\zeta)) < M_1$ for $\zeta \in \overline{\Omega}_1$. By continuity there is a simply connected domain Δ_1 , $\Omega_1 \subset \subset \Delta_1 \subset \subset \Delta$, such that $\rho(f(\zeta)) < M_1$ for $\zeta \in \overline{\Delta}_1$. Let $U_0 = \emptyset$ and for $n \in \mathbb{N}$ denote by U_n the sublevel set $\{z \in X; \rho(z) < M_n\}$. Since M_n is a regular value of ρ it holds that $\overline{U}_n = \{z \in X; \rho(z) \leq M_n\}$. This implies that \overline{U}_n is a holomorphically convex compact set, because on a Stein manifold plurisubharmonic hull equals holomorphic hull.

We shall construct inductively a decreasing sequence of domains $\{\Delta_n\}$ conformally equivalent to Δ , an increasing sequence of domains $\{\Omega_n\}$ conformally equivalent to Δ , $\Omega_n \subset \subset \Delta_n$ ($n \in \mathbb{N}$), a sequence

of continuous maps $g_n: \overline{\Delta}_n \rightarrow X$, holomorphic on Δ_n , and a decreasing sequence of positive numbers $\{\epsilon_n\}$, satisfying for each $n \in \mathbb{N}$ the following

- (I) $g_n(\zeta) \in U_n \setminus (\overline{U}_{n-1} \cup Y)$ ($\zeta \in \overline{\Delta}_n \setminus \overline{\Omega}_n$),
- (II) $g_{n+1}(\zeta) \notin \overline{U}_{n-1} \cup Y$ ($\zeta \in \overline{\Delta}_{n+1} \setminus \overline{\Omega}_n$),
- (III) $d(g_n(\zeta), g_{n+1}(\zeta)) < \frac{\epsilon_n}{2^n}$ ($\zeta \in \Omega_n$),
- (IV) $g_{n+1}(0) = g_n(0)$ and $g'_{n+1}(0) = g'_n(0)$,
- (V) if $z \in X$ such that $d(z, g_n(\overline{\Delta}_n \setminus V)) < \epsilon_n$ then $z \in U_n \setminus Y$,
- (VI) if $z \in X$ such that $d(z, g_{n+1}(\overline{\Delta}_{n+1} \setminus \overline{\Omega}_n)) < \epsilon_{n+1}$ then $z \notin \overline{U}_{n-1}$.

Let $g_1 = f$ and let Δ_1 and Ω_1 as above. Then (I) holds. Choose ϵ_1 , $0 < \epsilon_1 < \epsilon$, so small that (V) holds for $n = 1$. Suppose that $j \in \mathbb{N}$ and that we have constructed g_n , Δ_n , Ω_n and ϵ_n , $1 \leq n \leq j$, such that (I) and (V) hold for $1 \leq n \leq j$ and (II), (III), (IV) and (VI) hold for $1 \leq n \leq j - 1$. It follows by Lemma 2.2 that the set $(\overline{U}_{j-1} \cup Y) \cap \overline{U}_{j+1}$ is holomorphically convex. Therefore there is a smooth strongly plurisubharmonic exhaustion function ρ_{j+1} on X such that $\rho_{j+1}(z) < 0$ ($z \in (\overline{U}_{j-1} \cup Y) \cap \overline{U}_{j+1}$) and $\rho_{j+1}(g_j(\zeta)) > 0$ ($\zeta \in \overline{\Delta}_j \setminus \overline{\Omega}_j$) [Hör, Theorem 5.1.6]. There is N so big that $U_j \subset \{z; \rho_{j+1}(z) < N\}$. We use Theorem 2.1 to get a continuous map $g_{j+1}: \overline{\Delta}_j \rightarrow X$, holomorphic on Δ_j , with the following properties

- (a) $\rho_{j+1}(g_{j+1}(\zeta)) > N$ for $\zeta \in b\Delta_j$,
- (b) $\rho_{j+1}(g_{j+1}(\zeta)) > 0$ for $\zeta \in \overline{\Delta}_j \setminus \overline{\Omega}_j$,
- (c) $d(g_{j+1}(\zeta), g_j(\zeta)) < \frac{\epsilon_j}{2^j}$ for $\zeta \in \Omega_j$, and
- (d) $g_{j+1}(0) = g_j(0)$ and $g'_{j+1}(0) = g'_j(0)$.

By (a) and by the choice of N we get that $\rho(g_{j+1}(\zeta)) > M_j$ ($\zeta \in b\Delta_j$). Thus there is M , $M_j < M < M_{j+1}$, such that the holomorphic disc $g_{j+1}(\Delta_j)$ and the level set $\{z; \rho(z) = M\}$ intersect transversally. It follows by (c) and (V) that $g_{j+1}(\Omega_j \setminus V) \subset U_j$. This and the fact that $\rho \circ g_{j+1}$ is subharmonic imply that there is a simply connected component of $\{\zeta \in \Delta_j; \rho(g_{j+1}(\zeta)) < M\}$ which contains Ω_j . We denote such component by Δ_{j+1} . Choose a simply connected domain $\overline{\Omega}_{j+1}$, $\Omega_j \subset \subset \Omega_{j+1} \subset \subset \Delta_{j+1}$, such that $\rho(g_{j+1}(\zeta)) > M_j$ ($\zeta \in \overline{\Delta}_{j+1} \setminus \overline{\Omega}_{j+1}$). It is easy to see that Δ_{j+1} , Ω_{j+1} , g_{j+1} satisfy (I) for $n = j + 1$ and (II), (III) and (IV) for $n = j$. Since we have $g_{j+1}(\overline{\Delta}_{j+1}) \subset U_{j+1}$ and $g_{j+1}(\overline{\Delta}_{j+1} \setminus \overline{\Omega}_j) \cap Y = \emptyset$ and since by (V) for $n = j$ and by (c) we get that $g_{j+1}(\overline{\Omega}_j \setminus V) \cap Y = \emptyset$ it follows that $g_{j+1}(\overline{\Delta}_{j+1} \setminus V) \subset U_{j+1} \setminus Y$.

This together with (II) for $n = j$ implies that there is ϵ_{j+1} , $0 < \epsilon_{j+1} < \epsilon_j$, so small that (V) holds for $n = j + 1$ and that (VI) holds for $n = j$. The construction is finished.

Denote by Ω the union $\cup_{j=1}^{\infty} \Omega_j$. As Ω is a union of an increasing sequence of simply connected open sets it is simply connected and therefore conformally equivalent to the unit disc. It follows by (III) that for $\zeta \in \Omega$ the sequence $g_n(\zeta)$ is Cauchy with respect to the complete metric d therefore it converges to $g(\zeta)$. Since the convergence is uniform on compact sets it follows that the map g is holomorphic on Ω .

Next we show that the map g and the domain Ω have all the required properties. Fix $j \in \mathbb{N} \cup \{0\}$. It follows by (III) that

$$\begin{aligned} d(g(\zeta), g_{j+1}(\zeta)) &\leq d(g_{j+1}(\zeta), g_{j+2}(\zeta)) + d(g_{j+2}(\zeta), g_{j+3}(\zeta)) + \cdots < \\ &< \frac{\epsilon_{j+1}}{2^{j+1}} + \frac{\epsilon_{j+2}}{2^{j+2}} + \cdots < \epsilon_{j+1} \quad (\zeta \in \Omega_{j+1}). \end{aligned} \quad (1)$$

Thus for $\zeta \in \Omega_{j+1} \setminus \Omega_j$ it holds by (VI) that $g(\zeta) \notin U_{j-1}$. This implies that $g: \Omega \rightarrow X$ is a proper map. To prove that $g(\Omega \setminus V)$ avoids Y , choose $\zeta \in \Omega \setminus V$. There is $j \in \mathbb{N}$ so large that $\zeta \in \Omega_{j+1}$. It follows by (1) and by (V) that $g(\zeta) \notin Y$. By (1) for $j = 0$ we obtain that $d(g(\zeta), f(\zeta)) < \epsilon$ for $(\zeta \in V)$. We get by (IV) that $g(0) = f(0)$ and $g'(0) = f'(0)$. This completes the proof. \square

Proof of Theorem 1.2. There are r and R , $0 < r < R < 1$, such that $V \subset\subset r\Delta$. One can choose ϵ_0 , $0 < \epsilon_0 < \epsilon$, so small that

$$\text{for } z \in X \text{ such that } d(z, f(r\Delta \setminus V)) < \epsilon_0 \text{ it holds that } z \notin Y. \quad (2)$$

There is $\delta > 0$ so small that

$$V \subset\subset (r - \delta)\Delta \subset (r + \delta)\Delta \subset\subset R\Delta, \quad (3)$$

$$\zeta \in r\Delta, \zeta' \in \Delta \text{ such that } |\zeta - \zeta'| < \delta \text{ then } d(f(\zeta), f(\zeta')) < \frac{\epsilon_0}{2}. \quad (4)$$

Let $\Omega_0 = \emptyset$ and choose an increasing sequence $\{R_n\}$ of positive numbers converging to 1 with $R_1 > R$. We shall construct inductively an increasing sequence of simply connected domains $\{\Omega_n\}$ such that $R_n\Delta \cup \Omega_{n-1} \subset\subset \Omega_n \subset\subset \Delta$, a decreasing sequence of positive numbers $\{\epsilon_n\}$, $\epsilon_1 < \frac{\epsilon_0}{2}$, and a sequence of proper holomorphic maps $g_n: \Omega_n \rightarrow X$ such that

- (a) $g_n(\zeta) \notin Y$ for $\zeta \in \Omega_n \setminus V$,
- (b) $d(g_n(\zeta), f(\zeta)) < \epsilon_n$ for $\zeta \in R_n\Delta \cup \Omega_{n-1}$,

(c) $g_n(0) = f(0)$ and $g'_n(0) = f'(0)$.

Assume that we have already constructed Ω_n and ϵ_n ($0 \leq n \leq j$) and g_n ($1 \leq n \leq j$) for some $j \in \mathbb{N} \cup \{0\}$. One can choose ϵ_{j+1} , $0 < \epsilon_{j+1} < \frac{\epsilon_j}{2}$, with the following property

for $z \in X$ such that $d(z, f((R_{j+1}\Delta \cup \Omega_j) \setminus V)) < \epsilon_{j+1}$ it holds that $z \notin Y$. (5)

Using Lemma 2.3 for $V = R_{j+1}\Delta \cup \Omega_j$ and $\epsilon = \epsilon_{j+1}$ we obtain a simply connected domain Ω_{j+1} , $R_{j+1}\Delta \cup \Omega_j \subset\subset \Omega_{j+1} \subset\subset \Delta$, and a proper holomorphic map $g_{j+1}: \Omega_{j+1} \rightarrow X$ which satisfy (b) and (c) for $n = j + 1$ and it holds that $g_{j+1}(\zeta) \notin Y$ for $\zeta \in \Omega_{j+1} \setminus (R_{j+1}\Delta \cup \Omega_j)$. By (5) and (b) we get $g_{j+1}(\zeta) \notin Y$ for $\zeta \in (R_{j+1}\Delta \cup \Omega_j) \setminus V$, thus (a) holds for $n = j + 1$. This completes the construction.

Note that $\cup_n \Omega_n = \Delta$. Caratheodory kernel theorem [Car, Pom] implies that the sequence of conformal maps $h_n: \Delta \rightarrow \Omega_n$, such that $h_n(0) = 0$, $h'_n(0) > 0$, converges uniformly on compact sets to identity. Choose n so big that

$$|h_n(\zeta) - \zeta| < \delta \quad (\zeta \in r\Delta). \quad (6)$$

Let $g = g_n \circ h_n$. By the above $g: \Delta \rightarrow X$ is a proper holomorphic map and (c) implies (iii). Take $\zeta \in r\Delta$. By (6) and (3) we get that $h_n(\zeta) \in R\Delta$ which by (b) implies that $d(g_n(h_n(\zeta)), f(h_n(\zeta))) < \frac{\epsilon_0}{2}$. It follows by (6) and (4) that $d(f(h_n(\zeta)), f(\zeta)) < \frac{\epsilon_0}{2}$. Therefore $d(g(\zeta), f(\zeta)) < \epsilon_0$ ($\zeta \in r\Delta$). This proves (ii) and for $\zeta \in r\Delta \setminus V$ this together with (2) implies that $g(\zeta) \notin Y$. Choose $\zeta \in \Delta \setminus r\Delta$. By (6) it follows from Rouché's theorem that $(r - \delta)\Delta \subset h_n(r\Delta)$ and thus we get by (3) that $h_n(\zeta) \notin V$. By (a) it follows that $g(\zeta) \notin Y$. This proves (i). The proof is complete. □

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